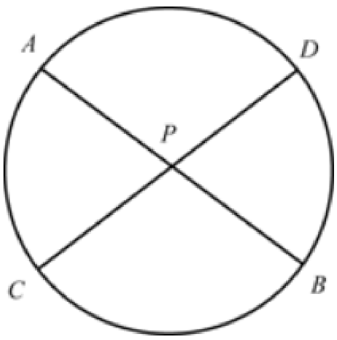
1



a $AP{\cdot}PB = CP{\cdot}PD$

$5 \times 4 = 2PD$

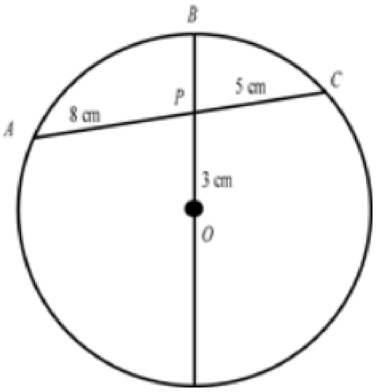
$PD = 10 \text{ cm}$

b $AP{\cdot}PB = CP{\cdot}PD$

$4PB = 3 \times 8$

$PB = 6 \text{ cm}$

2



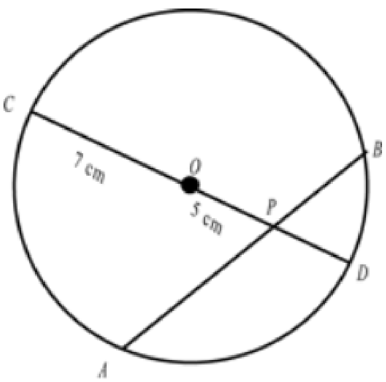
Let the centre of the circle be O and the length of the radius r cm.

Extend OP to meet the circumference of the circle at C and D.

$CP = r - 3$ and $PD = r + 3$

$$CP \cdot PD = AP \cdot PB$$
$$(r-3)(r+3) = 8 \times 5$$
$$r^2 - 9 = 40$$
$$r^2 = 49$$
$$r = 7 \text{ cm}$$

3



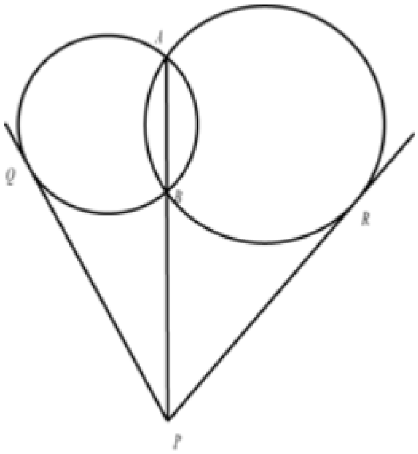
$PD = 7 \text{ cm} - 5 \text{ cm} = 2 \text{ cm}$

Let $PB = x \text{ cm}$

$PA = 4x \text{ cm}$

$$AP \cdot PB = CP \cdot PD$$
$$4x \times x = 12 \times 2$$
$$x^2 = 6$$
$$x = \sqrt{6}$$
$$\therefore \quad AB = 4\sqrt{6} + \sqrt{5}$$
$$= 5\sqrt{6} \text{ cm}$$

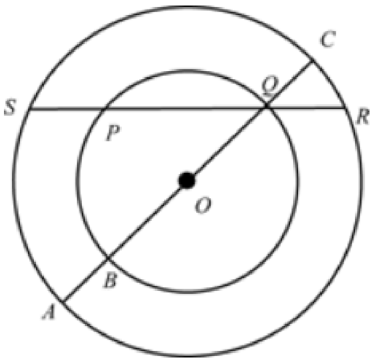
4



Use theorem 9:

$$PQ^2 = PA \cdot PB$$
$$PR^2 = PA \cdot PB$$
$$\therefore \quad PQ^2 = PR^2$$
$$PQ = PR$$

5



Let the centre of the circles be O.

Let the radii of the larger and smaller circles be R and r respectively.

Let QP produced meet the larger circle at S.

By symmetry, $SP = RQ$.

Extend OQ to meet the larger circle at A and C, and the smaller circle at B.

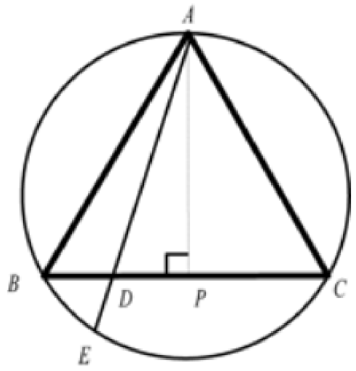
Since $SP = RQ$,

$$SP + PQ = RQ + PQ$$
$$\therefore \quad SQ = PR$$

Using the large circle,

$$SQ \cdot RQ = AQ \cdot CQ$$
$$PR \cdot RQ = (R + r)(R - r), \text{ which is constant}$$

6



Let P be a point on BC such that AP is perpendicular to BC.

Because ABC is isosceles, AP will bisect AB. Let $AP = x$ and $PC = PB = y$.

$DP = y - BD$

$CD = 2y - BD$

Using Pythagoras' theorem twice, we get $AB^2 = x^2 + y^2$ in triangle ABP and in triangle ADP.

$$\begin{aligned} AD^2 &= x^2 + (y - BD)^2 \\ &= x^2 + y^2 - 2y \times BD + BD^2 \\ &= AB^2 - BD(2y - BD) \\ &= AB^2 - BD \cdot CD \end{aligned}$$

$BD \cdot CD = DEAD$

$$\begin{aligned} \therefore\quad AD^2 &= AB^2 - DE \cdot AD \\ AB^2 &= AD^2 + DE \cdot AD \\ &= AD(AD + DE) \\ &= AD \cdot AE \end{aligned}$$